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Atmospheric Dispersion of a Wide Bandwidth Radar Signal Designed for Pulse Compression

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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LINCOLN LABORATORY

ATMOSPHERIC DISPERSION OF A WIDE BANDWIDTH RADAR SIGNAL DESIGNED FOR PULSE COMPRESSION

S. L. BORISON

Group 41

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LEXINGTON MASSACHUSETTS

Abstract

Dispersion of a wide-bandwidth radar signal by a static atmosphere is considered. Using experimental data to fit the attenuation (in db/km) due to molecular resonances, the index of refraction may be inferred. The dispersive delay due to two-way propagation through the atmosphere is then compared to the dispersion one would normally design for a pulse compression receiver. Comparison indicates that atmospheric dispersion is quite small unless the center frequency is near the lowest water vapor resonance at λ = 1.35 cm.

Accepted for the Air Force
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Introduction

The dispersive properties of the atmosphere have been widely studied in the past. By the simple requirement of causality, Kramers and Kronig showed that the real and imaginary parts of the complex dielectric constant are Hilbert transforms of each other. Using these relations, Van Vleck[1] showed that a very small change in the real part of the dielectric constant over the microwave region will automatically imply a serious attenuation of microwaves. In fact, he showed that a change of one part in 10^7 between $\lambda = \infty$ and $\lambda = 1$ cm implies at least .5 db/km attenuation for all wavelengths shorter than about 2 cm. He comments, "Thus, we can safely conclude that for all frequencies in the microwave region, the static value of the dielectric constant can be used without appreciable error."

It is, in fact, the purpose of this paper to investigate the effects of such a small dielectric gradient (with respect to frequency) on a wide band-width signal which propagates through a static atmosphere. At the outset, it will be well to realize that we shall only obtain an estimate of such effects. Basically, this is due to the fact that it is very difficult to obtain accurate theoretical or experimental values of the complex dielectric constant except near a microwave resonance of atmospheric constituents such as molecular oxygen and water vapor.

In Section I we briefly review the description of waves propagating in an homogeneous, isotropic medium of complex dielectric constant. In Section II the transmitted radar waveform is introduced by means of spectral weighting. The effects of scattering are mentioned and a discussion of the radar receiver enables us to define the dispersive time delay of a pulse compression system. This delay function of frequency enables one to make a comparison with the dispersion introduced by propagation through the atmosphere.

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1. For a general reference for material contained in this paper, see Propagation of Short Radio Waves, M.I.T. Rad. Lab. Series No. 13, Ch. 8, McGraw-Hill (1951).

In Section III, the dispersion and attenuation of the atmosphere is assumed to be a small effect near the radar center frequency, f_o. Appropriate expansions of the dielectric constant about f_o, and exponential scaling with altitude allows one to define the dispersion and attenuation constants in terms of which one may calculate these effects. In Section IV we use the general resonance formulae given by Van Vleck[1] and the experimental and theoretical data to determine the dispersion and attenuation constants. Finally, the dispersive delay is compared to that which one would normally implement for a linear frequency-modulated (fm) signal, and it is shown that atmospheric dispersion of an X-band signal of duration .1-μsec and 1-GC bandwidth is negligible. In fact, dispersion of a short-duration, wide-bandwidth signal by the atmosphere is unimportant for frequencies sufficiently below the first water vapor resonance.

I. Plane Waves in a Medium of Complex Dielectric Constant

Although the description of a plane wave propagating in an homogeneous isotropic medium of complex dielectric constant is well known, we include a few of the basic equations. The wave equation for a one-dimensional harmonic field is

$$(\nabla^2 + \epsilon k^2)\,\vec{E}_\omega(\vec{r}) = 0 \quad , \tag{1.1}$$

where $k = \omega/c$; ω is the radian frequency, c is the velocity light, and ϵ is the complex dielectric constant at the frequency $f = \omega/2\pi$. The solutions of this equation are

$$\vec{E}_\omega(\vec{r}) = \vec{E}_o\, e^{i(n + i\gamma)\,\vec{k}\cdot\vec{r}} \quad , \tag{1.2}$$

where $|\vec{k}| = k$ and $\vec{k}\cdot\vec{E}_o = 0$. The constants n and γ are determined by

$$(n + i\gamma)^2 = \epsilon_1 + i\epsilon_2 \quad , \tag{1.3}$$

where ϵ_1 and ϵ_2 are the real and imaginary parts of the dielectric constant, respectively. Solving for n and γ , we find (choosing the least attenuated solutions)

$$n^2 = \frac{1}{2}\left[\epsilon_1 + \sqrt{\epsilon_1^{\ 2} + \epsilon_2^{\ 2}}\right] \quad , \tag{1.4}$$

and

$$\gamma^2 = \frac{1}{2}\left[-\epsilon_1 + \sqrt{\epsilon_1^{\ 2} + \epsilon_2^{\ 2}}\right] \quad . \tag{1.5}$$

If we expand these solutions, assuming $\epsilon_2/\epsilon_1 << 1$ (in practice this is quite valid), we find

$$n \simeq \sqrt{\epsilon_1} \quad , \tag{1.6}$$

and

$$\gamma \simeq \frac{1}{2}\epsilon_2 \quad . \tag{1.7}$$

This finally yields the solution which propagates with the least attenuation,

$$\vec{E}_\omega(\vec{r}, t) = \vec{E}_o e^{i(\sqrt{\epsilon_1}\, \vec{k}\cdot\vec{r} - \omega t)} e^{-\frac{1}{2}\epsilon_2\, \vec{k}\cdot\vec{r}} \quad . \tag{1.8}$$

II. Radar Transmission and Reception

Suppose we choose our radar coordinate system such that $\vec{k}$ is along the x-axis; then

$$\vec{E}_\omega(x, t) = \vec{E}_o e^{i(\sqrt{\epsilon_1}\, kx - \omega t)} e^{-\frac{1}{2}\epsilon_2\, kx} \quad . \tag{2.1}$$

In practice, of course, we shall transmit a wave train of some finite duration. This wave train is then simply represented by

$$\vec{E}(x, t) = \int_{-\infty}^{\infty} d\omega\, S(\omega)\, \vec{E}_\omega(x, t) \quad , \tag{2.2}$$

where $S(\omega)$ indicates the spectral content of the wave. We now assume that this wave travels out from the radar a distance ℓ and is reflected by a stationary point scatterer (independent of polarization and frequency). The scattered field at the radar is then

$$\vec{E}_s(o,t) = \frac{A}{\sqrt{4\pi}\,\ell^2} \int_{-\infty}^{\infty} d\omega S(\omega) \vec{E}_o e^{i(2\sqrt{\epsilon_1}\,k\ell - \omega t)} e^{-\epsilon_2 k\ell} \quad , \qquad (2.3)$$

where A is the complex radar scattering amplitude of the point scatterer (the radar cross section, σ, is given by $\sigma = |A|^2$). Furthermore, we may assume that the radar is a linear device such that the signal received within the radar is given by

$$s(t) = \frac{A}{\sqrt{4\pi}\,\ell^2} \int_{-\infty}^{\infty} d\omega \vec{E}_o \cdot \vec{R}(\omega) S(\omega)\, e^{i(2\sqrt{\epsilon_1}\,k\ell - \omega t)} e^{-\epsilon_2 k\ell} \quad , \qquad (2.4)$$

where $\vec{R}(\omega)$ is the polarization dependent gain of the system at the frequency, f. (Note that one could easily include polarization and frequency dependence due to the scatterer by simply replacing A inside the integral by $\overleftrightarrow{A}(\omega)$, the scattering dyadic, or scattering matrix; i.e., use the combination $\vec{E}_o \cdot \overleftrightarrow{A}(\omega)$ in Eqs. (2.3) and (2.4). Furthermore, one could also include the effects of a moving target by simply replacing ω by $\omega(1 + \frac{2v}{c})$ in the ωt phase term, where v is the velocity component of the scatterer toward the radar.)

The polarization and frequency dependence of $\vec{R}(\omega)$ will normally separate such that we may assume

$$\vec{R}(\omega) = \hat{\epsilon}_R R(\omega) \qquad (2.5)$$

where $\hat{\epsilon}_R$ is a unit vector. The function $R(\omega)$ will normally be the function matched to $S(\omega)$; i.e., $R(\omega)$ will be the filter designed to maximize the received signal energy-to-noise power ratio (for white noise)[2]. Thus, we may assume that

$$R(\omega) = S^*(\omega) \quad . \qquad (2.6)$$

(Note that there may also be some spectral weighting to reduce range sidelobes.) Although it is difficult to make any general statements regarding the properties of $S(\omega)$, i.e., the best wide-bandwidth signal to provide good pulse compression with very low range sidelobes, one may consider the behavior of

2. See, for example, Skolnik, Introduction to Radar Systems, Ch. 9, McGraw-Hill (1962).

the first frequency derivative of the phase of $S(\omega)$; this function is called the dispersive time delay[3]. If

$$S(\omega) = | S(\omega) | e^{i\psi(\omega)} \quad ,$$

then

$$T(f) = \frac{d\psi(\omega)}{d\omega} \quad , \tag{2.7}$$

and its general behavior is indicated in Fig. 1, where T is the transmitted pulse length, W is the instantaneous frequency bandwidth, and f_o is the center frequency. (A constant delay has been neglected.)

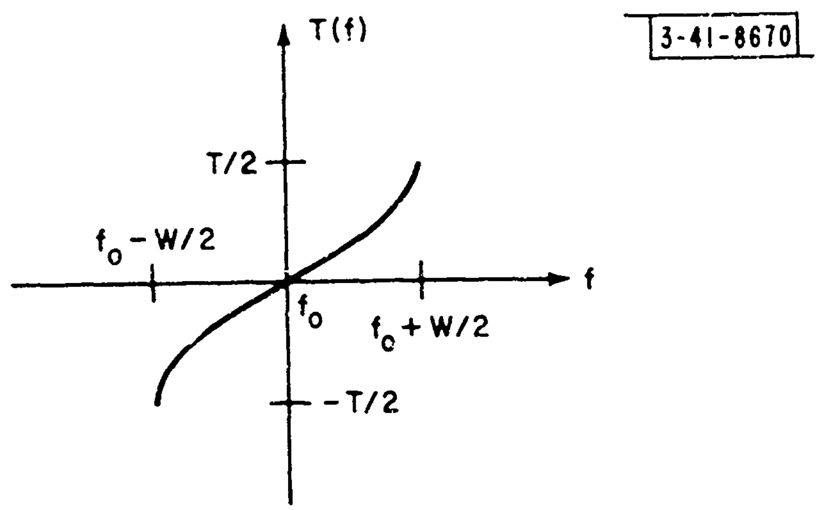

Fig. 1. General Dispersive Time Delay versus Frequency

A useful example is the linear fm signal, for which a good approximation to $T(f)$ is

$$T_{\ell fm}(f) \simeq \frac{Tf_o}{W}\left(\frac{f - f_o}{f_o}\right) \quad . \tag{2.8}$$

3. See, for example, E. N. Fowle, "The Design of FM Pulse Compression Signals," IEEE Trans. on Inf. Theory, pp. 61-67 (1964).

III. Dispersion and Attenuation Coefficients

Having introduced the concept of the dispersive time delay for a radar receiver, we are now able to make a meaningful evaluation of the effects of atmospheric dispersion. In particular, we may compare the dispersive time delay introduced via propagation in the atmosphere with that of a pulse compression receiver. Simultaneously, we may also estimate the attenuation induced by the atmosphere.

Equation (2.4) indicates the dispersion and attenuation due to a single atmospheric constituent. Since these effects in the microwave region are predominantly due to the resonances of water vapor and molecular oxygen, we must generalize to include two constituents. Furthermore, we must account for the exponential atmosphere and, in fact, allow for different scale heights for each constituent. This might seem impossible at first since the index of refraction is not a linear function of ϵ_1 (see Eq. (1.6)) ; however, we may limit our interest to carrier frequencies which are many "resonant widths" lower than the lowest resonance (the water vapor resonance at 1.35 cm). If we expand the complex dielectric constant as a function of frequency about the radar center frequency, f_o , then as a function of altitude, h , above the earth's surface,

$$\epsilon_1(f,h) = \epsilon_1(f_o, h) + \left(\frac{f-f_o}{f_o}\right) \sum_i K_i(f_o, h) + \frac{1}{2}\left(\frac{f-f_o}{f_o}\right)^2 \sum_{ij} K_{ij}(f_o, h) + \ldots, \tag{3.1}$$

and

$$\epsilon_2(f, h) = \epsilon_2(f_o, h) + \left(\frac{f-f_o}{f_o}\right) \sum_i \xi_i(f_o, h) + \frac{1}{2}\left(\frac{f-f_o}{f_o}\right)^2 \sum_{ij} \xi_{ij}(f_o, h) + \ldots, \tag{3.2}$$

where $K_i(f_o, h)$ and $\xi_i(f_o, h)$ are due to the i-th constituent. Similarly, $K_{ij}(f_o, h)$ and $\xi_{ij}(f_o, h)$ are functions due i, j constituent mixing, such that $i \neq j$ accounts for interactions between pairs of constituents. Higher-order coefficients may be similarly interpreted as being due to higher-order interactions. We shall assume that only the linear frequency dependence is important and neglect all higher-order terms. Furthermore, we shall use the expansion

$$\sqrt{\epsilon_1(f)} = \sqrt{\epsilon_1(f_o, h)}\ \left[1 + \frac{\sum_i K_i(f_o, h)}{2\epsilon_1(f_o, h)}\left(\frac{f-f_o}{f_o}\right)\right] \quad . \tag{3.3}$$

We may now assume that

$$\epsilon_1(f_o, h) = 1 + \Delta\epsilon_1(f_o)e^{-h/H_o} \quad ,$$

$$\epsilon_2(f_o, h) = \epsilon_2(f_o)e^{-h/H_o} \quad ,$$

$$K_i(f_o, h) = K_i(f_o)e^{-h/H_i} \quad ,$$

and

$$\xi_i(f_o, h) = \xi_i(f_o)e^{-h/H_i} \quad , \tag{3.4}$$

where H_o is the average scale height for the atmosphere and the H_i , K_i , and ξ_i are the scale height, dispersion coefficient,and attenuation coefficient for the i-th constituents contributing to the linear frequency dependence of $\epsilon(f)$ near $f = f_o$.

Finally, the attenuation and dispersion phase terms in Eqs. (2.3) and (2.4) are the result of evaluating the phase-path integral

$$L = 2\int_o^{\ell} dx\ [\sqrt{\epsilon_1} + \frac{i}{2}\epsilon_2] \quad . \tag{3.5}$$

If we now include the dependence of ϵ_1 and ϵ_2 on altitude, and assume a flat earth geometry in the vicinity of the radar (see Fig. 2), we use $h = x \sin\theta$ to find

$$L(\ell, \theta) = 2\int_o^{\ell} dx\ \{[1 + \frac{1}{2}\Delta\epsilon_1(f_o)e^{-\frac{x\sin\theta}{H_o}}][1 + \frac{1}{2}(\frac{f-f_o}{f_o})\sum_i K_i(f_o)e^{-\frac{x\sin\theta}{H_i}}]$$

$$+ \frac{i}{2}[\epsilon_2(f_o)e^{-\frac{x\sin\theta}{H_o}} + (\frac{f-f_o}{f_o})\sum_i \xi_i(f_o)e^{-\frac{x\sin\theta}{H_i}}]\} \quad . \tag{3.6}$$

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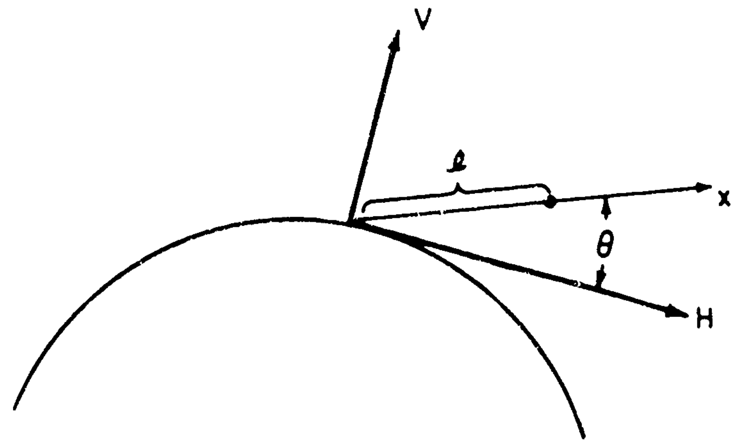

Fig. 2. Radar Coordinate System

It should be noted that $L(\ell, \theta)$ is not valid for $\theta = 0$ due to the flat earth approximation. In fact, we must restrict θ such that (R_E is the radius of the earth)

$$\theta > H_o/R_E \simeq 1/10 \text{ degree.}$$

Equation (2.4) is then generalized to yield

$$s(t) = \frac{A\,\vec{E}_o \cdot \hat{\varepsilon}_R}{\sqrt{4\pi}\;\ell^2} \int_{-\infty}^{\infty} d\omega\, R(\omega)\, S(\omega)\, e^{i\,[kL(\ell,\theta) - \omega t]} \quad . \tag{3.7}$$

In order to obtain simple answers, let us specify two extreme cases of observation: (1) Vertical observation with $\ell \gg H_o$; (2) Horizontal observation with $\ell \ll R_E$. For these two cases we obtain (neglecting terms of order $\Delta\varepsilon_1(f_o)\,K_i(f_o)$)

$$L_V \simeq 2\ell\,[1 + \tfrac{1}{2}\Delta\varepsilon_1(f_o)\frac{H_o}{\ell}] + (\frac{f-f_o}{f_o})\sum_i K_i H_i$$

$$+ iH_o[\varepsilon_2(f_o) + (\frac{f-f_o}{f_o})\sum_i \xi_i(f_o) H_i/H_o] \quad , \tag{3.8}$$

and

$$L_H \simeq 2\ell\,[1 + \tfrac{1}{2}\Delta\epsilon_1(f_o)] + (\frac{f-f_o}{f_o})\,\ell \sum_i K_i\,(f_o)$$

$$+ i\ell\,[\epsilon_2(f_o) + (\frac{f-f_o}{f_o}) \sum_i \xi_i(f_o)] \quad . \tag{3.9}$$

Using Eqs. (3.8) and (3.9) we may now define the vertical and horizontal dispersive delay and attenuation factors, respectively, as

$$\Delta T_V(f) = L_V/c - \frac{2\ell}{c} \quad , \tag{3.10}$$

$$\Delta T_H(f) = L_H/c - \frac{2\ell}{c} \quad , \tag{3.11}$$

$$A_V(f) = e^{-kH_o[\epsilon_2(f_o) + (\frac{f-f_o}{f_o})\sum_i \frac{H_i}{H_o}\xi_i(f_o)]} \quad , \tag{3.12}$$

and

$$A_H(f) = e^{-k\ell\,[\epsilon_2(f_o) + (\frac{f-f_o}{f_o})\sum_i \xi_i(f_o)]} \quad . \tag{3.13}$$

Note that the dispersive delay times contain an average correction to the usual gross range approximation; e.g., in ΔT_H it is $(\frac{\ell}{c})\,\Delta\epsilon_1(f_o)$.

IV. Evaluation of the Dispersion and Attenuation Coefficients of Water Vapor and Molecular Oxygen

In order to calculate the dispersive time delay and attenuation factors, we must now evaluate the set of constants introduced in the preceding discussion. The quantum mechanical theory of molecular electric and magnetic resonance phenomena is the basis for theoretical evaluations and much detailed work has been reported in the literature. Analysis of a single resonance yields the results, valid near the i-th resonance[1],

$$\epsilon_{1i}(f) = 1 + \frac{A_i f_i}{2}\left[\frac{(f_i - f)}{\Delta f^2 + (f - f_i)^2}\right] \quad , \tag{4.1}$$

and

$$\epsilon_{2i}(f) = \frac{f_i}{2}\left[\frac{\Delta f_i}{\Delta f_i^2 + (f - f_i)^2}\right] \quad , \tag{4.2}$$

where A_i is a dimensionless constant, f_i is the frequency at the center of the i-th resonance, and Δf_i is the line-breadth constant. We see that $[\epsilon_1(f) - 1]$ achieves the maximum and minimum values of $\pm A_i f_i / 4\Delta f_i$ at $f = f_i \mp \Delta f_i$, respectively; thus, the maximum change in $\epsilon_1(f)$ occurs in this frequency interval and has the value $\Delta\epsilon_1 = A_i f_i / 2\Delta f_i$. Figure 3 (taken from Ref. 1) indicates the behavior of $\epsilon_{1i}(f)$ and the attenuation in db/km (see Eq. 4.6) near the resonance.

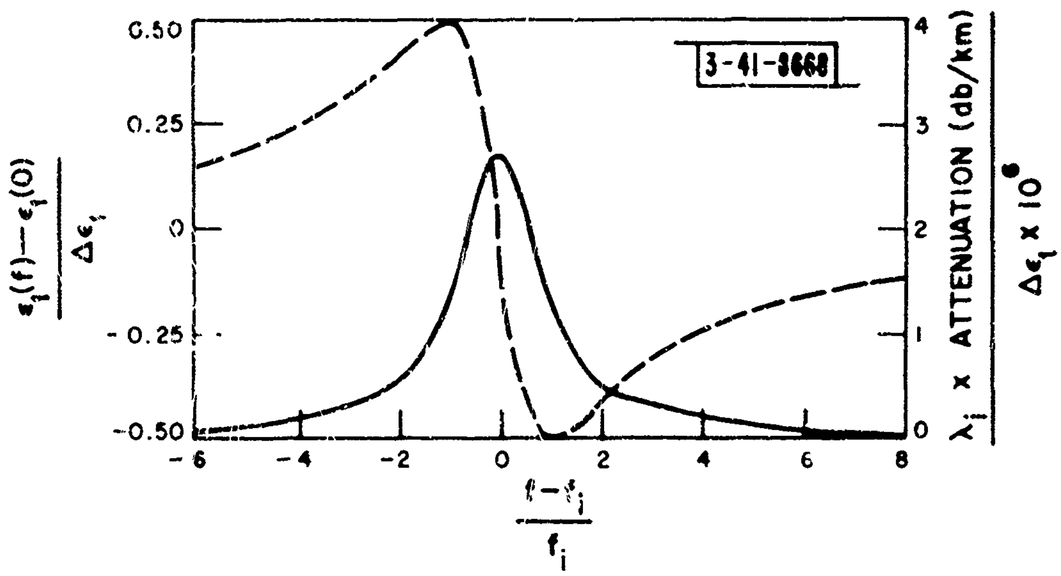

Fig. 3. Dependence of Resonant Absorption and Refraction on Frequency in the Vicinity of Resonance

Experimental verification of Eq. (4.1) has always been a difficult, if not impossible, task. For a narrow-bandwidth system one must be able to perform precise, unambiguous phase measurements on a scattered wave -- and one must also theoretically subtract out all other possible phase deviations. Thus, one usually finds in the literature experimental verifications of Eq. (4.2) by means of the i-th attenuation factor. In this way the two parameters, A_i and Δf_i, may be evaluated and one may then assume that they are the appropriate parameters for Eq. (4.1). This is, in fact, what we shall assume in order to evaluate the dispersion constant, K_i.

Using Eqs. (4.1) and (4.2) for a single resonance, where A_i and Δf_i have been evaluated by comparison with experiment, we may evaluate $K_i(f_o)$ and $\xi_i(f_o)$ by expanding $\epsilon_1(f)$ and $\epsilon_2(f)$ about $f = f_o$ and making the appropriate comparisons with Eqs. (3.1) and (3.2) combined with Eq. (3.4). Following this procedure we find (where we have assumed that $(f_i - f_o) \gg \Delta f_i$)

$$K_i(f_o) = \frac{A_i f_i f_o}{2(f_i - f_o)^2} , \tag{4.3}$$

and

$$\xi_i(f_o) = \frac{A_i f_i \Delta f_i}{(f_i - f_o)^3} . \tag{4.4}$$

We have discussed the case of a single molecular resonance which implies Eqs. (4.1) and (4.2); in reality we must deal with multiple resonances. In that case it is not true that we may simply compute $\epsilon(f)$ by adding individual contributions since there will certainly be interactions. In particular, collision broadening may occur. We shall assume that the A_i and Δf_i are not appreciably affected if the resonances are sufficiently well separated. Thus, we shall assume that

$$\epsilon_2(f) \simeq \sum_i \frac{A_i f_i}{2} \left[\frac{\Delta f_i}{\Delta f_i^2 + (f - f_i)^2} \right] . \tag{4.5}$$

Using Eq. (4.5) we may compute the attenuation in db/km to find

$$A_{db/km}(f) = \frac{2\pi\, \epsilon_2(f)}{\lambda}\, 10^6 \log e \quad , \tag{4.6}$$

where λ is measured in cm. Figures 4 and 5, taken from Ref. 1, indicate a combination of experimental and theoretical results obtained for the first oxygen and water vapor resonances.

Referring to Eqs. (4.3) and (4.4), it is obvious that the resonances nearest to the radar center frequency are important. In principle, we evaluate $A_{db/km}(f)$ at all f_i. Equation (4.6) then gives a set of coupled linear equations in the A_i. We shall here assume that the resonances are sufficiently separated that the overlap of the tails may be neglected as a first approximation. Thus, we simply measure each A_i separately with the relation

$$A_i = \frac{\lambda_i^2 (\Delta f_i / c)}{\pi\, 10^6 \log e}\, A_{db/km}(f_i) \quad , \tag{4.7}$$

where λ_i is in cm and $(\Delta f_i/c)$ is in cm^{-1}. Note that the attenuation due to water vapor depends linearly on the density, ρ, which may vary by a large amount. For temperate latitudes ($20^{\circ}C$) in the summer, the average density is about 7.5 gm/m^3. On the other hand, at saturation at $20^{\circ}C$, sea level, ρ = 17. gm/m^3, and under tropical conditions the content can be even higher. We shall evaluate A_i/ρ for the water vapor resonances.

Since the second oxygen attenuation is small, and it occurs at a smaller wavelength with a much narrower width, Eq. (4.7) indicates that A_i for the second resonance is very small compared to that for the first resonance. Furthermore, since the dispersive effects at f_o are reduced by $\frac{1}{(f_i - f_o)^2}$, we may calculate a first approximation to the dispersion by using only the first of the water vapor resonances. We may also note that this approximation will indicate a minimum estimate of the dispersion since all K_i for the higher resonances are positive if f_o is less than all the f_i. The results obtained are

$$A_{O_2} = 1.1 \times 10^{-7} \qquad \text{at } \lambda = .5 \text{ cm} \quad ,$$

and

$$A_{H_2O}/\rho = 2.9 \times 10^{-9} \qquad \text{at } \lambda = 1.35 \text{ cm} \quad . \tag{4.8}$$

Finally, we may use the results stated in Eq. (4.8) in conjunction with Eq. (4.3) to calculate the dispersion constants at the frequency f_o. With these constants we may then evaluate the dispersive time delay of Eqs. (3.10) or (3.11). As an example, we evaluate ΔT_H to find

$$\Delta T_H = \left(\frac{f-f_o}{f_o}\right)\frac{\ell}{c}\left\{\frac{1.1 \times 10^{-7}(6 \times 10^{10} f_o)}{2(6 \times 10^{10} - f_o)^2} + \frac{2.9 \times 10^{-9}\rho(2.2 \times 10^{10} f_o)}{2(2.2 \times 10^{10} - f_o)^2}\right\} . \tag{4.9}$$

In order to compare this result with the dispersion characteristic of a linear fm pulse compression receiver (see Eq. (2.8)), we shall assume the example $f_o = 10$ Gc , $W = 1$ Gc , $T = .1$ μsec , $\rho = 7.5$ gm/m^3; thus

$$\frac{\Delta T_H}{T_{\ell fm}} \simeq 3 \times 10^{-8}\left(\frac{\ell}{c}\right)\left(\frac{W}{Tf_o}\right) = 10^{-7}\,\ell \quad ,$$

where ℓ is in km. Even if $\ell = R_E \simeq 6000$ km , the effect is negligible for a ten percent bandwidth at X-band. In fact, if T is fixed at .1 μsec, $W = f_o/10$, and $\ell = 10$ km, the average dispersion of a wide-bandwidth signal by the atmosphere is only ten percent of the receiver dispersion when one uses a carrier frequency which is about 100 Mc lower than the first water vapor resonance at 22 Gc.

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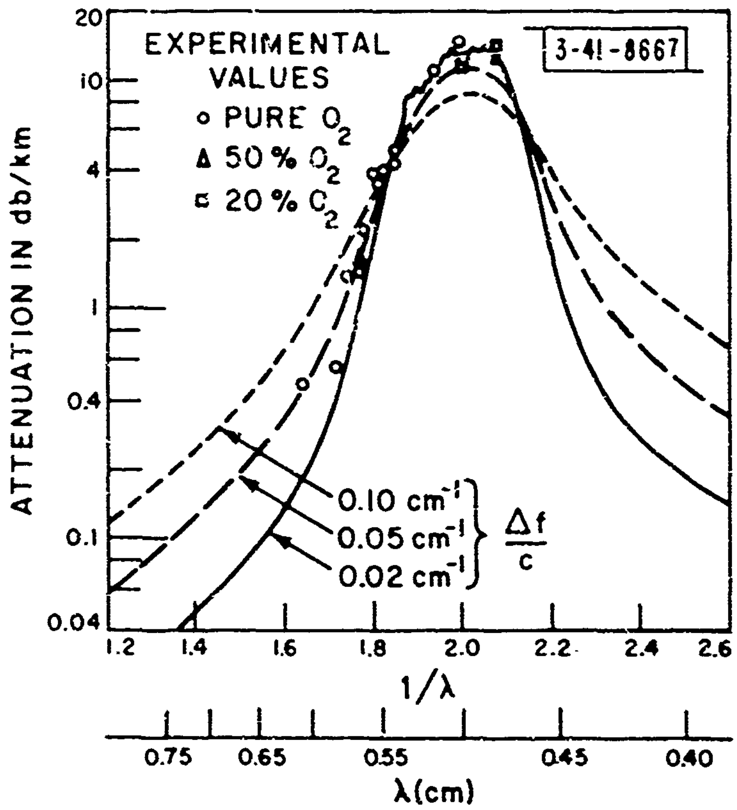

Fig. 4. Atmospheric Attenuation by Oxygen as a Function of Wavelength

The measurements, when made on other than 20% O_2, are reduced to "air equivalent values" by assuming that at given total pressure the absorption is proportional to the partial pressure of oxygen.

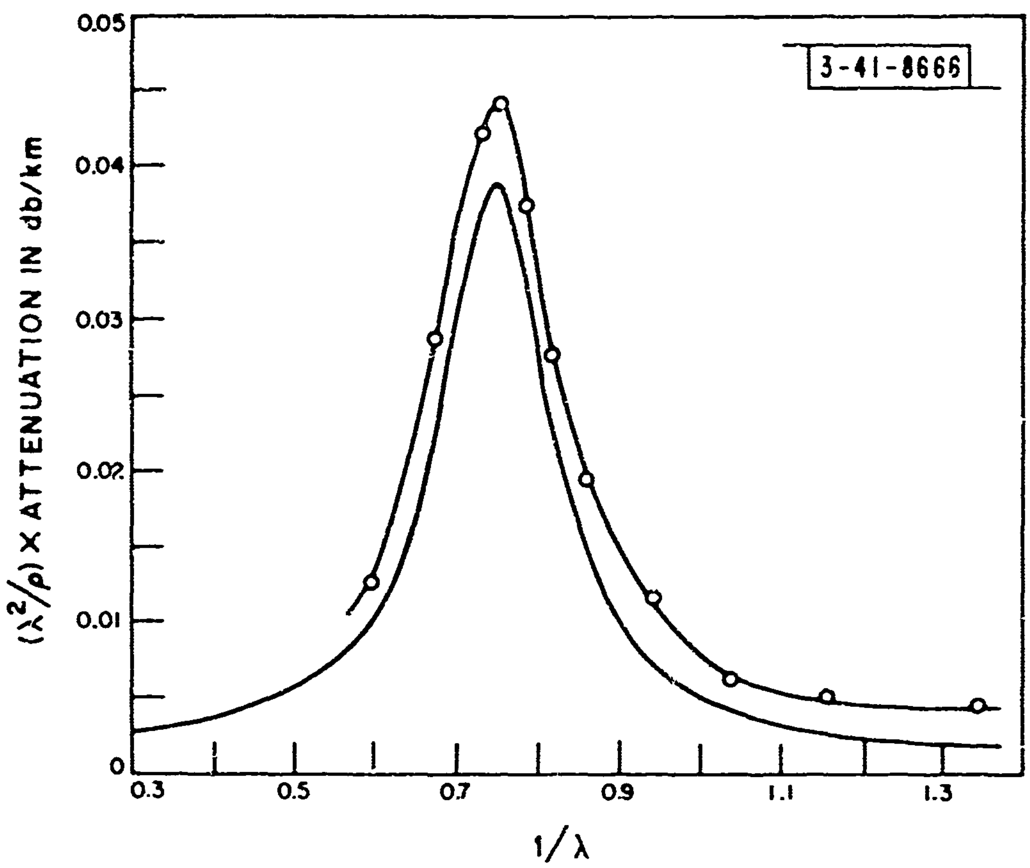

Fig. 5. Theoretical and Experimental Results on Attenuation by Water Vapor in the 1-cm Region at a Temperature of 45° C.

The lower curve gives theoretical values based on $\Delta f/c = 0.087\ \mathrm{cm}^{-1}$. The upper curve represents measurements made at the Columbia Radiation Laboratory. The units used are: λ in cm, and ρ in grams per cubic meter.